Algebra II Curriculum Guide
Tier 1 & 2

Unit 3: Exponential and Logarithmic Relationships
February 1 – April 12

ORANGE PUBLIC SCHOOLS 2018-2019

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

Contents

Unit Overview

Unit 3: Rational Functions and Equations
Overview
This course uses Agile Mind as its primary resource, which can be accessed at the following URL: ➢ www.orange.agilemind.com Each unit consists of 1-3 topics. Within each topic, there are "Exploring" lessons with accompanying activity sheets, practice, and assessments. The curriculum guide provides an analysis of teach topic, detailing the standards, objectives, skills, and concepts to be covered. In addition, it will provide suggestions for pacing, sequence, and emphasis of the content provided.
Essential Questions
➢ How do you identify an arithmetic series and a geometric series? ➢ How do you write a function rule for arithmetic series and geometric series? ➢ What is an exponential function? ➢ What are the key characteristics of the parent graph of the parent graph of an exponential function? ➢ How do you use transformation to graph exponential function? ➢ How are exponential growth/decay used to solve real world problems? ➢ What is a logarithmic function? ➢ What are the characteristics of the parent graph of a logarithmic function? ➢ How do you use transformations to sketch graphs of logarithmic functions? ➢ How do you evaluate logarithmic functions? ➢ How do you solve exponential equations? ➢ How do you solve logarithmic equations?
Enduring Understandings
➢ In an arithmetic series, each value in the series differs from its predecessor by the same amount and in geometric series, the ratio of each term to its predecessor is a constant. ➢ In arithmetic series the function rule is linear and in a geometric series the ➢ function rule is exponential ➢ All arithmetic and geometric sequences can be expressed recursively and explicitly. ➢ Exponential function is expressed in the form of f(x) = ab^{x}, where b is the constant multiplier and greater than 1, a is the initial value and x is the exponent. ➢ Exponential value shows rapid growth or rapid decay ➢ Parent graph of the exponential function has a domain of $(-\infty, \infty)$ and range of $(0, \infty)$, no extrema or symmetry, has an y intercept of (0,1) and the function is increasing. ➢ Exponential growth model are used mostly for population growth and compound interest. ➢ Exponential Decay model are used for half-life of chemical compound and also for population that is decreasing. ➢ Logarithmic function is the inverse of exponential function and has the form of $log_b y = x$, where b is the base and greater than 1, y is the argument and x is the exponent. ➢ In order to evaluate logarithmic function you need to find inverse. ➢ In order to solve exponential and logarithmic equation you need to need find inverse.

Common Core State Standards

1) A.SSE.4 : Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.
2) A-CED.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3) F.IF.3: Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.
4) F.BF.1: Write a function that describes a relationship between two quantities.
   a. Determine an explicit expression, a recursive process, ~~or steps for calculation from a context.~~
5) F.BF.2: Write Arithmetic and Geometric sequence both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
6) F.LE.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
7) F.IF.1: *Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of is the graph of the equation y = f(x)*
8) F.IF.2: *Use function notation. Evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.*
9) F.IF.5: *Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*
10) A-CED A.2: equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

11) F-BF.5: (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents

12) A.SSE.3c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15t can be rewritten as $(1.151/12)^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%

13) F-IF B.4: Interpret functions that arise in applications in terms of the context. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; ~~relative maximums and minimums~~; ~~symmetries;~~ end behavior; and ~~periodicity.~~

14) F-IF B.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

F-BF.3: Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

15) F-IF.7e. Graph exponential and logarithmic functions, showing intercepts and end behavior, ~~and trigonometric functions, showing period, midline, and amplitude~~

16) F-IF.8b: Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10,}$ and classify them as representing exponential growth or decay

17) F-LE A.1.c: Distinguish between situations that can be modeled with ~~linear functions~~ and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

18) F-LE A.2: Construct ~~linear~~ and exponential functions, including ~~arithmetic and geometric sequences,~~ given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

19) F-LE A.3: Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing ~~linearly, quadratically, or (more generally) as a polynomial function.~~

20) F-LE A.4: Construct and compare linear, quadratic, and exponential models and solve problems 4.For exponential models, express as a logarithm the solution to abct = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology

21) F-LE A5. Interpret the parameters in a[n] ~~linear~~ or exponential function in terms of a context

22) A-CED. A.1: Create equations that describe numbers or relationships 1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

23) A.RE.D.11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Major Content
Supporting Content
Additional Content
~~Parts of standard not contained in this unit~~
Algebra I Content

Algebra II Unit 3

Unit 3: Sequence and series, exponential function, Logarithmic function(35 Days)

- Arithmetic and geometric sequences and series
- Exponential functions
- Logarithmic functions
- Using Logarithms to solve equations and ~~inequalities~~
- Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Big Rock CCSS	Related Topic	Lesson Objective	Notes
F.IF.3: Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.	Arithmetic sequence and series	Lesson 1.1 • Recognize arithmetic sequences • Identify the common difference. Recognize geometric sequences and identify the common ratio	
F.IF.3 F.BF.1: Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, ~~or steps for calculation from a context~~ F.BF.2: Write Arithmetic and ~~Geometric~~ sequence ~~both~~ recursively and with an explicit formula, use them to model situations, and translate between the two forms.	Geometric sequence and series	Lesson 1.2 • Recognize that sequences are functions with integer domains. • Use function notation to write recursive rule for an arithmetic sequence • Use the explicit expression to find a future term in a sequence • Use the concept of sequence to solve real life problem	
F.IF.3, F.BF.1, F.BF.2 F.LE.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).		Lesson 2.1 • Write the function for Geometric sequence Lesson 2.2 • Use the geometric sequence concept to solve real life problem	
A.SSE.4 : Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. Clarification: Students will not have to derive the formula in PARCC but they will have to use it		Lesson 2.3 • Derive the Geometric series • Use Geometric Series to solve problems Note: Derive the formula for the series with students before using it.	
F.LE.2: Construct ~~linear~~ and exponential functions, including ~~arithmetic~~ and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	Exponential Functions	Lesson 3.1A (F.LE.2, A.SSE.1, F.IF.8b) Using the concept of Geometric Sequence SWBAT • Construct an exponential function and use it to solve problems	

A.SSE.1 Interpret expressions that represent a quantity in terms of its context.* F.IF.8b: Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = $(1.02)^t$, y = $(0.97)^t$, y = $(1.01)12^t$, y = $(1.2)^t/10$, and classify them as representing exponential growth or decay. F.IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and ~~sketch graphs showing key features given a verbal description of the relationship~~. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.* F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context. N. RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents A.SSE.3c Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest ate if the annual rate is 15%.		• Interpret parts of expressions in terms of its context (growth/decay factor and initial amount) • Identify exponential growth /decay from the context • Identify exponential growth and decay from the function and graph 3.1b (F.LE.2, F.IF.4, F.LE.5) Using graphs, function and table SWBAT • Identify appropriate values for growth/decay factor. • Interpret the graph and expression in terms of context • Identify and Interpret the domain range and end behavior of the exponential function Lesson 3.2a (N.RN.2, A.SSE.3c) • Transform Exponential Functions using properties of exponents • Rewrite expressions using properties of exponents. i.e Using the properties of exponents, re-write exponential functions based on the structure of expression(NO context) Lesson 3.2b (A.SSE.3c) • Transform Exponential Functions using properties of exponents I.e. Using the properties of exponents, re-write exponential functions based on the context For example: • Convert annual interest rate quarterly, monthly And use it to solve problem half life…..(Refer to PARCC 2016 question 18)	

F-BF.5: (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents F-LE A.4: For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology. .	Logarithmic Expressions/Function	Lesson 4.1 (F-LE.4, F.BF.5) Use the concept of inverse relations SWBAT • understand logarithmic function is the inverse of exponential function and has the form of $log_b y = x$, where b is the base and greater than 1, y is the argument and x is the exponent • Evaluate logarithmic function using technology Lesson4.2 (F.LE.4) • Apply the properties of exponents to derive log sum and log difference property and use these two properties to solve problems • Develop log exponentially property and use it to solve problem Lesson 4.3 (F.LE.4) • Develop log change base property And use it to solve problem • Strategically use properties of log to solve problems.	

Calendar

February 2019						
Sun	Mon	Tue	Wed	Thu	Fri	Sat
					1	2
3	*4*	5	6	7	8	9
10	11	12	13	14	15	16
17	18 Winter Break No SCHOOL	19 Winter Break No SCHOOL	20 Winter Break No SCHOOL	21 Winter Break No SCHOOL	22 Winter Break No SCHOOL	23
24	25	26	27	28		

March 2019						
Sun	**Mon**	**Tue**	**Wed**	**Thu**	**Fri**	**Sat**
					1	2
3	*4*	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

April 2019						
Sun	**Mon**	**Tue**	**Wed**	**Thu**	**Fri**	**Sat**
	1	2	3	4	5	6
7	*8*	9	10	11	12	13
14	15	6	17	18	19	20
21	22	23	24	25	26	27
28	29	29				

Assessment Framework

Assessment	Assignment Type	Grading	Source	Estimated in-class time	When?
Diagnostic Assessment *Unit 3 Diagnostic*	Test	Traditional (zero weight)	Curriculum Dept. created – see Dropbox	< ½ block	Beginning of unit
Benchmark 1 Assessment	Test	Mostly Online graded Some questions need rubrics	Edulastic	Yes	End of MP 3
Teacher Created Assessments	Test	Traditional	Teacher Created	1 block	In between topics
Performance Task *Unit 4 Performance Task1*	Authentic Assessment	Rubric	Topic constructed response (also see Dropbox)	½ block	In topic 8
Performance Task *Unit 4 Performance Task2*	Authentic Assessment	Rubric	Topic constructed response (also see Dropbox)	½ block	In topic 9
Quizzes	Quiz	Rubric or Traditional	Teacher created or “Practice” in Agile Minds	< ½ block	Varies (must have 3 quizzes per MP)

Algebra II Unit 3

Scope and Sequence

Overview			
Topic	Name	Agile Mind "Blocks"*	Suggesting Pacing
1	**Arithmetic and Geometric Sequence and Series**	**4**	**5 Days**
12	**Exponential Functions**	**7**	**10 days with the supplements**
13	**Logarithmic Function**	**8**	**8 Days**
14	**Using Logarithmic to solve equations**	**6**	**5 Days**

Diagnostic Assessment	**½ day**
Benchmark Assessment for MP3	**1 Day**
Teacher created assessment	**2 days**
Performance Task 1	**½ day**
Performance Task 2	**½ day**
Review	**1 day**
Quizes	**1 to 2 days**
Total	**33.5 days**

*1 Agile Mind Block = 45 minutes

Topic 1: Arithmetic and Geometric series and sequences
Topic Objectives (Note: these are not in 3-part or SMART objective format) 1. Recognize arithmetic sequences and identify the common difference. 2. Recognize geometric sequences and identify the common ratio. 3. Recognize that sequences are functions with integer domains. 4. Find explicit terms and sums for arithmetic and geometric sequences. 5. Determine convergence and divergence of infinite series 6. Learn the conditions for convergence of infinite geometric series 7. Use explicit formulas to find the sum of an infinite geometric series. Focused Mathematical Practices • MP 2: Reason abstractly and quantitatively • MP 3: Construct viable arguments and critique the reasoning of others • MP 5: Use appropriate tools strategically • MP 6: Attend to precision • MP 7: Look for and make use of structure • MP 8: Express regularity in repeated reasoning Vocabulary Arithmetic sequence, common difference, geometric sequence, common ratio, recursive definition, arithmetic series, geometric series, indices, sigma, finite series, infinite series, partial sums, convergent, and divergent. Fluency ✓ Graphing equations ✓ Using function notation ✓ Solving Literal equations (cx + a = d) ✓ Identifying rate of change from tables, graphs and equations

Suggested Topic Structure and Pacing

Block	Objective(s) covered	Agile Mind "Blocks" (see Professional Support for further lesson details)	MP	Additional Notes
1	1,2 and part of 4	*Block 1* *Block 2*	2,3, 5, 8	Sequence and series were introduced to students in Algebra I _Topic 3_explore_function notation from page 7 – 10. Same example of congruent triangles was used
2-3	3,4	*Block 3* *Block 4* *Block 5*	2, 4, 5, 8	Students will be introduced to summation notation for the first time. You need to spend extra time on having student practice with sigma notation.
4	1,2,3,4	*Block 6* *Block 7*	5, 4, 7, 8	Block 7 : students completing one of the constructed responses; this is optional/flexible.

Algebra II Unit 3

CCSS	Concepts What students will know	Skills What students will be able to do	Material/Resource
A.SSE.4 : Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. A-CED.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	***Block 1: Review*** • Function definition, different types of function, Domain and range, patterns, rate of change, slope intercept form of an equation, point slope form of an equation, and perimeter of any shapes ***Block 1: New*** • Sequence and term. Relating terms to its previous term using function notation • Definition of series as the sum of terms of a sequence • Recursive form and it's limitation • Formula for arithmetic sequence • Importance of arithmetic sequence	**Block 1 Review** • Evaluate functions Identify rate of change from graphs, tables and equation, Write slope intercept of an equation from graph and table. ***Block 1: New*** • Identifying arithmetic sequence through the common difference • Identify geometric sequence through common ratio • Write arithmetic sequence	Agile Mind Topic 1 *** Overview** *** Exploring** "Arithmetic sequence and series" **P 1-7** Suggested assignment: SAS 1 Q4a-e SAS 2 Q4a-d and 7
F.IF.3: Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1. F.BF.1: Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, ~~or steps for calculation from a context.~~ F.BF.2: Write Arithmetic and Geometric sequence both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	***Block 2 : Review*** Exponential equation, percent ratios, arithmetic sequence, subscript notations, expressions and functions with exponents ***Block 2: New*** • Continue to explore arithmetic sequence • Sigma notation as the sum of terms • Definition of geometric sequence. • formula for geometric series ***Block 3 : Review*** Percent, plotting points, geometric series, evaluating expressions with exponents ***Block 3 : New*** • Concept of limits, concept of infinite series, solving problems with infinite series, concept of convergence and divergence of infinite series	***Block 2 : Review*** • Write exponential equations • Solve problems with percent, evaluate functions with exponents ***Block 2: New*** • Continue to write arithmetic sequence rules • Derive the formula for arithmetic sequence • Use Sigma notation as the sum of terms • Complete the arithmetic sequence from a table • Derive the formula for geometric series **Block 3 : Review** • Solving problems with Percent. • plotting points, • Formula for geometric series, • Evaluating expressions with exponents ***Block 3 : New*** • solving problems with infinite series,	Agile Mind Topic ***Exploring** **"**Arithmetic sequence and series" **P 8-12** ***Exploring** "Geometry sequence and series" ***Exploring** "Infinite series" **More Practice:** P1-13 SAS 2: Q12a-b SAS 3:Q6a-c
F.LE.2: Construct linear and exponential	***Block 4:***	• Identifying convergence and divergence of infinite series	

functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	Review the concept and skill to complete Guided Practice and Constructed Response	***Block 4:*** Review the concept and skill to complete Guided Practice and Constructed Response	Agile Mind Topic 1 ***Guide Practice** ***Constructed** Response **(one-to-one computer needed)** **More Practice** P14-17

Topic 12: Exponential functions

Topic Objectives (Note: these are not in 3-part or SMART objective format)

1. Formulate exponential relationships written in recursive notation
2. Derive the formula for compound interest.
3. Calculate compound interest yearly, monthly, weekly, daily
4. Transform exponential function

Focused Mathematical Practices

- MP 2: Reason abstractly and quantitatively
- MP4: Model with mathematics
- MP 5: Use appropriate tools strategically
- MP 6: Attend to precision
- MP7: Look for and make sense of structure

Vocabulary
Exponential relationship, recursive notation, compound interest
Fluency

- Exponential function
- Inverse function
- Domain and Range

Suggested Topic Structure and Pacing

day	**Objective(s) covered**	**Agile Mind "Blocks"** (see Professional Support for further lesson details)	**MP**	**Additional Notes**
Day 1	1, 2, 3	*Block 3*	2,4,7	Review writing exponential function from table/graphs during "do now" Skip "overview" and Explore "Fit the Model" Explore: "e" page 1 - 6 Note: Derivation of e is not an algebra II Concept
Day 2	4	*Block 5*	2, 4, 7	Explore: "Transforming exponential function" pages 1 - 8

CCSS	**Concepts** What students will know	**Skills** What students will be able to do	Material/Resource
1) F-LE A.2: Construct ~~linear and~~ exponential functions, ~~including arithmetic and~~ geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). 2) F-IF.8b: Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10,}$ and classify them as representing exponential growth or decay 3) A.SSE.3c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%	***Day 1*** ***Review*** • Definition of exponential function • Recursive model for exponential function • Concept of percent ***New*** • Definition of compound interest • Developing formula for compound interest with different frequencies	***Day 1*** ***Review*** • Writing exponential function • Evaluating problems with precents ***New*** • Find compound interest with different frequencies	***Day 1*** Agile Mind Topic 12 **P 1 - 4** *** Exploring** "e" **P 1 – 6** Suggested assignment: *More practice* p1-6 SAS 3 Q3 and 12
1) F-BF B.3 Build new functions from existing functions 3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for	***Day 2 (concept)*** ***Review*** • Exponential functions • Transformation ***New*** • Concept of transformation be used for exponential function just like any other functions	***Day 2 (skills)Review*** Transformation with any function ***New*** • Transformation of exponential functions	***Day 2 (Material)*** Agile Mind Topic 12 *** Exploring** * "Transforming exponential function" pages 1 - 8 Suggested assignment: SAS 4 Q14 – 17

Topic 13: Logarithmic function

Topic Objectives (Note: these are not in 3-part or SMART objective format)

1. Develop an understanding of the inverse relationship between exponential and logarithmic functions and convert back and forth between exponential and logarithmic functions.
2. Identify domain and range of exponential and logarithmic function with context and without context
3. Apply the prior knowledge of transformation to transform logarithmic functions
4. Apply properties of exponents to derive log sum and log difference property
5. Develop log exponential property
6. Define the term common logarithm and also develop the change of base formula

Focused Mathematical Practices

- MP 2: Reason abstractly and quantitatively
- MP4: Model with mathematics
- MP7: Look for and make sense of structure

Vocabulary: Rational Equations, Extraneous solutions

Fluency

- Exponential functions
- Properties of exponents
- Domain and range of a function
- Transformations of functions
- Inverse functions

Suggested Topic Structure and Pacing

Day	**Objective(s) covered**	**Agile Mind "Blocks"** (see Professional Support for further lesson details)	**MP**	**Additional Notes**
1	1	*Block 1*	4, 7	Overview: Page 1 – 4 Explore: Basics of Logarithms" Page 1 -4
2	2 and 3	*Block 2*		Explore: " Basics of Logarithms" Pages 5 – 14
3	4 & 5	*Block 3 and 4*		Explore: "Properties of Logarithm" Page 1 - 10
4	6	*Block 5 and 6*		Explore: "Natural log and change of base" Page 1 - 8

CCSS	Concepts What students will know	Skills What students will be able to do	Material/Resource
1) F-BF.5: (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents	***Day1 (Concept)*** ***Review:*** • Concept of inverse relations • Exponential function ***New*** • The term Log is used to write the inverse of the exponential function	***Day 1 (Skills)*** ***Review*** • Writing exponential function ***New*** • Convert exponential relationship with logarithmic relationship	Overview: **Page 1 – 4** ***Explore**: "Basics of Logarithms" **Page 1 -4** Suggested assignment: SAS 2 Q7a-c, 8a-c, and 9
1) F-IF B.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. 2) F-BF B.3 Build new functions from existing functions 3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them	***Day 2 (Concept)*** ***Review:*** • Definition Domain and range of any function • Transformation ***New*** • Parent logarithmic function	***Day 2 (Skills)*** ***Review*** • Finding domain and range of any function ***New*** • Finding domain and range of logarithmic function • Transformation of logarithmic function	***Explore**: "Basics of Logarithms" **Page 5-14** Suggested assignment: SAS 2 Q24-26, 27a-b, and 28 More practice p1-6

F-LE A.4: For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	***Day 3 (Concept)*** ***Review:*** • Properties of exponents ***New*** • Use the concept of properties of exponents to derive the Properties of Logarithms	***Day 3 (Skills)*** ***Review*** • Apply properties of logarithm ***New*** • Use properties of logarithm to re-write logarithmic expressions	***Explore:** "Properties of logarithms" **Page 1 - 10** Suggested assignment: SAS 3 Q6, 9, and 10a-c, and 15 More practice p7-12
F-LE A.4: For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	***Day 3 (Concept)*** ***Review:*** • Parent function of the logarithmic function ***New*** • Definition of natural log • Base 10	***Day 3 (Skills)*** ***Review*** • Log exponent property • Evaluating with logs ***New*** • Evaluating with natural log and base 10	***Explore:** "Properties of logarithm" **Page 1 - 10** Suggested assignment: SAS 4 Q2, 7a-d, and 8

Topic 14: Using logarithms to solve equations and inequality

Topic Objectives (Note: these are not in 3-part or SMART objective format)

After completing the topic square root functions and equations, students will be able to

1. Use tables and graphs to solve exponential equations
2. Solve exponential equations analytically/Algebraically

Focused Mathematical Practices

- MP 4: Model with mathematics
- MP 6: Attend to precision
- MP 7: Look for and make use of structure

Vocabulary

Quadratic formula, Imaginary numbers, complex numbers, discriminant, real roots and complex roots

Fluency

- Multiple solution strategies (tabular, graphical, analytic)
- Exponential and logarithmic functions

Suggested Topic Structure and Pacing

Day	**Objective(s) covered**	**Agile Mind "Blocks"** (see Professional Spport for further lesson details)	**MP**	**Additional Notes**
1	1	*Block 1*	4, 7	Overview Exploring " Tables and Graphs" page 1 – 6
2	3	*Block 2*		Exploring " Analytic Techniques" page 1 – 8
2		*Block 4*		"Constructed Response" SKIP INEQUALITY

CCSS	**Concepts** What students will know	**Skills** What students will be able to do	Material/Resource

1) A.RE.D.11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	***Day1 (Concept)*** ***Review*** • Multiple representations(tables and graphs) • Exponential functions ***New*** • Exponential equations	***Day 1 (Skills)*** ***Review*** • Using multiple representations to represent linear and quadratic functions ***New*** • Solving exponential equations using graphs and tables	**Day 1 (Material)** Agile Mind Topic 10 *** Overview** **Pgs. 1 -3** *** Exploring** "Tables and graphs " **Pgs. 1-6** SAS 2: Q2, 5, 6, 7
1) F-LE A.4: For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology. 2) A.RE.D.11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	***Day 2 (Concept)*** ***Review:*** • Systems of linear equations • Properties of logarithms ***New:*** • Understand that solution of a logarithmic equation can be in the same manner as another other equations	***Day 2 (Skills)*** ***Review*** • Solving systems of linear equations by substitution ***New*** • Solving exponential equations analytically	**Day 2 (Material)** Agile Mind Topic 10 *** Exploring** "Analytic techniques " P 1 – 8 Suggested assignment: SAS 3 Q4, 11a-c, 12a-c, and 13a-c

Supplemental Lessons

Lesson 3.1a

CCSS A.SSE.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

CCSS N.RN.2: Rewrite expressions involving radicals and rational exponents using the properties of exponents.

SMP: MP7

Evidence statement: (A.SSE.B)

Use the structure of polynomial, rational, or exponential expression to rewrite it, in a case where two or more rewriting steps are required.

* Tasks do not have context.

Lesson objectives:

* Rewrite expressions using properties of exponents

Task1:

Part A:

Do you think the following statements are true or false? Explain your reasoning. Talk to a partner.

$2^{2x} = 4^x$ $\quad$ $4^{2x} = 16^x$ $\quad$ $3^{3x} = 9^x$ $\quad$ $(\sqrt[4]{5})^x = 5^{x/4}$

Part B:

Which equations ae true for all values of x?

Select **all that apply** and showing your work and to **justify the answer(s) that you select.**

a. $3^{2-x} = 3^2 - 3^x$

b. $3^{x+2} = 9(3^x)$

c. $(3^x)^2 = (3^2)^x$

d. $9^{x+2} = 3^{2x+4}$

e. $27^{x} = (3^{x})^{3}$

f. $(\sqrt{5})(\sqrt[3]{5}) = 5^{\frac{5}{6}}$

Part C: Consider the original expression $3^{x} - 3^{x+2}$

Rewrite this expression as $a \cdot 3^{x}$

Guided Practice:

1. Solve each problem for x

   1) $4^{x} = 4^{2}$

   2) $4^{x} = 2^{b}$

   3) $32^{\frac{x}{3}} = 8^{x-12}$

2. For m>0, the expression $\frac{2(\sqrt{m})^{-2}}{\sqrt[4]{m}}$ can be rewritten in the form $2m^{a}$, where "a" is a fraction. What is the value of a?

3. In the equation $(5^{\frac{1}{2}})(5^{\frac{2}{3}})^{4} = 5^{x}$, what is the value of x?

Independent Practice:

1. An expression is given

$$\frac{(3x)}{(3x)^{\frac{2}{3}}}$$

If x > 0, which of the expression listed is equivalent to the expression given? Select all that apply.

a. $\frac{1}{3x}$ b. $\frac{1}{\sqrt{3x}}$ c. $\frac{1}{3\sqrt{x}}$ d. $(3x)^{1/2}$ e. $(3x)^{-1/2}$

2.

$$f(x) = \frac{(x^{-2})^3}{\left(x^{\frac{1}{4}}\right)^8}$$

If f(x) can be rewritten as x^a, what is the value of a ?

3. Rewrite the expression $\sqrt[3]{x^2}$ in exponential form.

4. Rewrite the expression $9\sqrt[5]{27}$ as a power of 3.

5. Which of the following is equivalent to $a^{\frac{1}{2}}b^{\frac{3}{4}}$?

a. $\sqrt{ab^3}$

b. $\sqrt{a^2b^3}$

c. $\sqrt[4]{ab^3}$

d. $\sqrt[4]{a^2b^3}$

Enrichment

A company that manufactures memory chips or digital cameras uses the formula

C=3$\sqrt{n}$ (40$\sqrt[6]{n}$ + 9$\sqrt[4]{n}$) to determine the cost, c, in dollars, o producing n chips. This formula can be written as c = 120$\sqrt[3]{n^a}$ + 27$\sqrt[4]{n^b}$, where a and b are constants. What ae the values of a and b?

Lesson 3.1 b

CCSS: A.SSE.3c: Use the properties of exponents to transform expressions or exponential functions. For example the expression 1.15t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%

Evidence statement:

*Tasks have a real-world context.
*The equivalent form must reveal something about the real-world context.
*Tasks require students to make the connection between the equivalent forms of the expression.
SMP: MP4, MP7

Task: 1.2b (CCSS: A.SSE.3c)

A bank provides a yearly compound interest proposal for student-saving account by using the formula

$f(t) = P(1.02)^t$ (P: the initial money deposited**; t: # of years**)

Eli wants to find a formula which compounds each month but will have the same balance by the end of each year in his account as using the bank yearly compound interest formula. He opened an account and deposited $500 in the account. He created the formula

$g(m) = 500(b)^m$ **(m: # of months)**

Eli thinks **$g(12) = f(1)$. What does that mean?**

Part A: Which of the following can be the value of b for monthly compound formula?

(a.) $b= 1.02$ (b.) $b= 1.02^{12}$ (c.) $b = 1.02^{1/12}$ (d.) $b = \frac{1.02}{12}$

Show your work to support the answer that you chose from above.

Part B: Which of the following expression can be used for Eli's formula g(m) ?

(a.) $g(m) = 500(1.02)^{12m}$ (b) $g(m) = 500(\frac{1.02}{12})^m$ (c) $g(m) = 500(1.02)^m$ (d) $g(m) =500(1.02)^{m/12}$

Show your work to support the answer that you chose from above.

Part C: Does the formula that you chose from Part B work for any year? Select a year to test your formula.

Part D: Based on the bank's formula derive a quarterly compound interest formula, k(q), for deposing $500 in a student saving account? (q: number of quarter years)

Guided Practice:

Question 1:

An analyst studying the population of a town determines that the population can be modeled by the formula $f(t)=120,000(1.015)^t$, where f(t) represents the population after t years.

A city council member makes this claim:

"Based on the formula, after 1 year the population will have increased by 1,800. Since 2,800 divided by 12 is 150, we can use the fact that the population increased by 150 people per month to predict the future population of the town".

Part A: Explain why the city council member's claim is or is not a valid way to predict the population.

Part B: Modify the initial given formula such that it represents the predicted population by month.

Part C: What is the population after 50 months?

Question 2:

The amount of a certain element that remains after *t* hours can be determined using the expression $3(0.6)^{\frac{t}{25}}$

Which of the following expression can be used to determine the element that remains after **n munities?**

(a) $3(0.6)^{60n/25}$ (b) $3(0.6)^{25n}$ (c) $3(0.6)^{n/25}$ (d) $3(0.6)^{n/60\bullet 25}$

Show your work to support your answer.

Independent Practice:

1. A city offered a program that provided incentives for residents who purchased energy-efficient appliances. During a certain year, participation in the program increased exponentially by 25 percent. What was the monthly rate of increase, expressed as a percent? Round your answer to one decimal place.

2. A bank pays depositors a 2% interest rate compounded semiannually. L P represents an initial deposit and let t represent the number of years that the deposit is in the bank. The expression $P(1+\frac{0.02}{2})^{2t}$ can be used to determine the account balance after t years. Which expression accurately reflects the annual interest rate?

(a) $P(1.01)^t$ (b) $P(1.21)^t$ (c) $P(1.0201)^t$ (d) $P(1.0404)^t$

Show your work to support your answer.

3. The population, in thousands, of a certain city can be modeled by the function $P(t) = 320(0.96)^{0.5t}$, where t is the number of years since 2000.

Part A: What was the population of the city in the year 2000?

Part B: Describe the rate of change of the city's population per year.

Enrichment material:

A scientist has a sample of bacteria that initially contains 10 million microbes. He observes the sample and finds that the number of bacterial microbes doubles every 20 minutes.

Part A: Write an exponential function that represents M, the total number of bacterial microbes in millions, as a function of t, the number of minutes the sample has been observed.

Part B: Determine how much time, to the nearest minute, will pass until there are 67 million bacterial microbes.

Ideal Math Block

The following outline is the department approved ideal math block for grades 9-12.

1) Do Now (7-10 min)
    a. Serves as review from last class' or of prerequisite material
    b. Provides multiple entry points so that it is accessible by all students and quickly scaffolds up

2) Task/Launch (10 – 15 min)
    a. Designed to introduce the lesson
    b. Uses concrete or pictorial examples
    c. Attempts to bridge the gap between grade level deficits and rigorous, on grade level content
    d. Provides multiple entry points so that it is accessible by all students and quickly scaffolds up

3) Mini-Lesson (15-20 min)
    a. Design varies based on content
    b. May include an investigative approach, direct instruction approach, whole class discussion led approach, etc.
    c. Includes CFU's
    d. Anticipates misconceptions and addresses common mistakes

4) Class Activity (25-30 min)
    a. Design varies based on content
    b. May include partner work, group work/project, experiments, investigations, game based activities, etc.

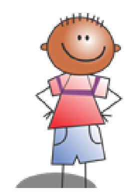
5) Independent Practice (7-10 min)
    a. Provides students an opportunity to work/think independently

6) Closure (5-10 min)
    a. Connects lesson/activities to big ideas
    b. Allows students to reflect and summarize what they have learned
    c. May occur after the activity or independent practice depending on the content and objective

7) DOL (5 min)
    a. Exit slip

MTSS MODEL

Whole Group Instruction	50 min	INSTRUCTION (Grades 9 – 12) Daily Routine: Mathematical Content or Language Routine Anchor Task: Anticipate, Monitor, Select, Sequence, Connect Collaborative Work* Guided Practice Independent Work (Demonstration of Student Thinking)	TOOLS Manipulatives RESOURCES Agile Mind	
Rotation Stations (Student Notebooks & Chromebooks Needed)	1-2X 25 min	STATION 1: Focus on current Grade Level Content STUDENT EXPLORATION* Independent or groups of 2-3 Emphasis on MP's 3, 6 (Reasoning and Precision) And MP's 1 & 4 (Problem Solving and Application) TOOLS/RESOURCES Agile Mind Math Journals	STATION 2: Focus on Student Needs TECH STATION Independent TOOLS/ RESOURCES Khan Academy Approved Digital Provider Fluency Practice	TEACHER STATION: Focus on Grade Level Content; heavily scaffolded to connect deficiencies TARGETED INSTRUCTION 4 – 5 Students TOOLS/ RESOURCES Agile Homework Manipulatives
	5 min	INSTRUCTION Exit Ticket (Demonstration of Student Thinking) TOOLS/RESOURCES Notebooks or Exit Ticket Slips		



Sample Lesson Plan (Agile Mind)

Lesson	Topic 12 Rational Functions Exploring “e”	**Days**	1
Objective	By studying a pattern in the table SWBAT 1. Derive the formula for compound interest. 2. Calculate compound interest yearly, monthly, weekly, daily	**CCSS**	F-LE A.2 F-IF.8b A.SSE.3c

Learning activities/strategies

Materials needed: Computer with projection device, graphing calculator

Fluency Practice: (5 minutes) If the sales tax in your city is 7.0%, and an item costs $144 before tax, how much tax would you pay on that item?

Do Now (5 minutes):

Red raised fire ants to study the population growth. During the first week, he started with 20 fire ants. The table shows his collections

Weeks	Fire ants
0	20
1	40
2	80
3	160
4	320

Write function relationships for the Fire ants with respect to the number of weeks.
Ask students to show the process from 0m weeks for every week. Discuss the pattern. Ask students what kind of function is it? And how do you know?

Starter/Launch (5 minutes):

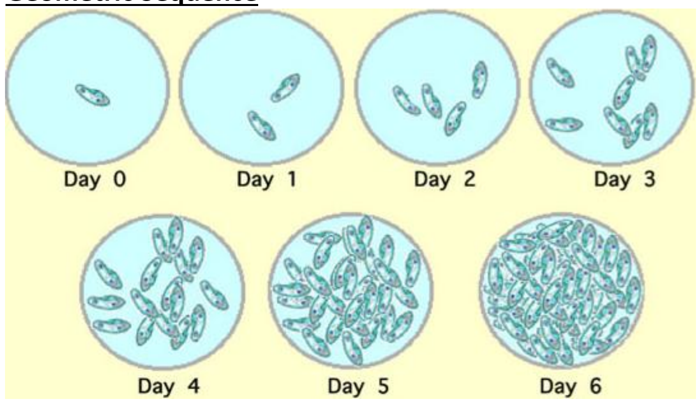
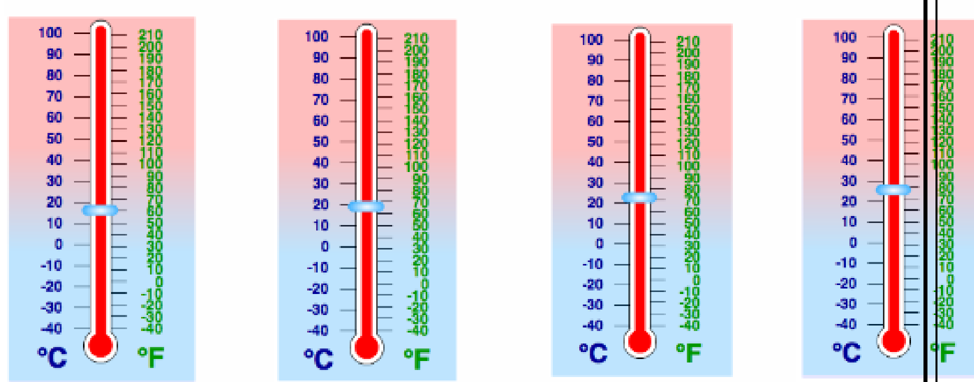
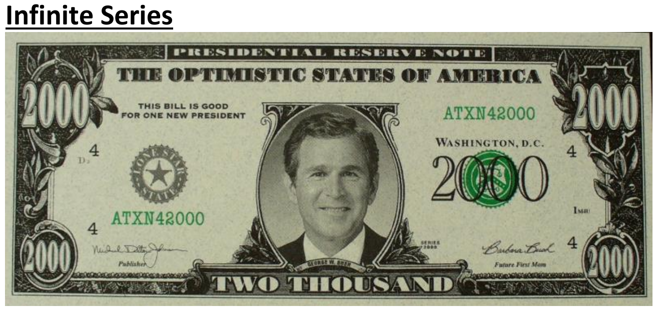
- Ask students think of any real life situation where exponential function can be applied (growth or decay) Introduce today’s objective

mini Lesson (20 minutes):

- Set up the scenario of Rachel's gift of $1000 from her grandmother on page 1 of the *Exploring* "*e*."
- Ask students “What would be the difference in depositing the $1000 into a savings account that pays 12% interest once per year and one that pays 1% interest every month?” Give students a minute or so to think about it on their own then share their thought with a partner.
- Ask students to find the amount of money in saving after 1 year for 1% interest every month. (Most students might do the recursive process just like page 2 of Exploring “e”
- Ask students to write the formula for recursive process by completing the puzzle on SAS 3 question # 2
- Ask students to work in air to answer the following question
  - What is a way to directly calculate the amount in the second savings account after 12 months? [SAS 3, question 4]
- If students struggle with the exponential expression, ask them to express the amount in the saving's account in month 3 using the recursive process and substitution in terms of A0. (So, A1 = A0 * 1.01, A2 = A1 * 1.01 = A0 * 1.01 * 1.01, A3 = A2 * 1.01 = A0 * 1.01 * 1.01 * 1.01, etc.)
- Remind students of the first savings account option and ask:
- Why does the account with the smaller interest rate end up with a larger balance? [SAS 3, question 5]
- Use page 5 to introduce the formula A = P(1 + r)t. [SAS 3, question 6] Spend some time making connections between A0 and P. Then make connections for students

	between the exponential function f(x) = a · bx and the compounding interest formula. Talk about the starting value and the constant multiplier. • NOTE: represent t is number of times interest rate is compounded instead of number of years **Group work/ Partner work (20 minutes)** Present the problem on page 5. Have students work in pairs or small groups to answer the questions. [SAS 3, questions 7-10] Debrief: Show the answers on page 5 for verification Derive the general formula for compound interest $p\left(1+\frac{r}{n}\right)$ **Independent Practice (12 minutes):** • Re-inforce SAS 3 Question 12 and 13 • Debrief and check for 2 minutes **Closure (2 minutes):** • Ask students what is compounding means, what does it mean to compound quarterly, monthly or yearly and which one is better. • Ask students hat each variable in compound interest represent. **DOL (5 minutes):**

Multiple Representations

Exponential function	
Verbal description	Exponential function is a function in the form of $y = ab^x$ in which the independent variable is the exponent, a is the initial value, and b is a constant
Real Life	**Geometric Sequence**  Changes in Paramecium over six days period is double from the preceding day.  Increasing the temperature of a cold room by 10% every hour follows a geometric sequence of 60, 66, 73, 80, 88….with a common ratio of 1.10 **Infinite Series** 

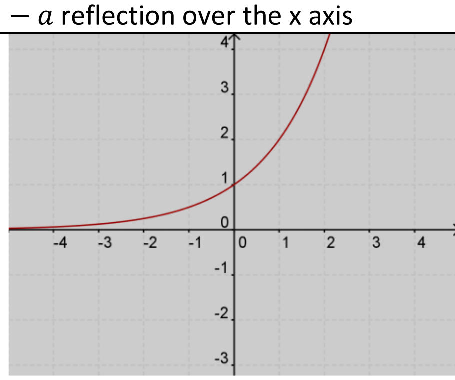
<table>
<tr><td></td><td>If you invest $2000 at the start of every year into an account earning 5% interest. Using the compound interest formula, we find that the value of the account at the end of the first 3 years is:

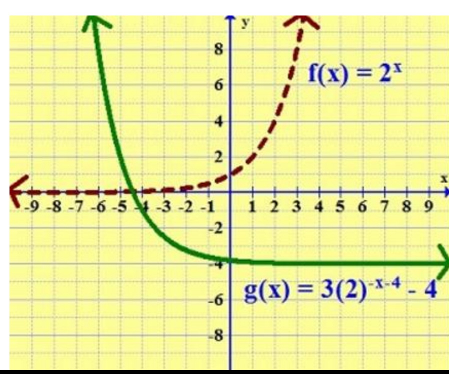
• Value at End of Year 1 = $2000 x 1.05^1= $2100

• Value at End of Year 2 = $2000 x 1.05^1 + $2000 x 1.05^2 = $4305

• Value at End of Year 3 = $2000 x 1.05^1 + $2000 x 1.05^2 + $2000 x 1.05^3 = $6620.25

If you look closely, you’ll see that this sequence of expressions for the value of the investment at the end of each year form a mathematical series where each term is the amount of money invested each year—$2000—multiplied by the interest rate raised to a higher and higher integer power</td></tr>
<tr><td>Function form</td><td>Parent Function   Where b is the base and x is the exponent
$f(x) = b^x$ b>0 and b ≠ 1

<u>Transformation</u>
Parent Function: $y = 2^x$
Transformed function $= -a(2^{x \pm h}) \pm k$
$+\,k$ moves up vertically
$-\,k$ moves down vertically
$+\,h$ moves to the right horizontally
$-\,h$ moves to the left horizontally
a>0 vertically stretches
a< 0 vertically shrinks
$-\,a$ reflection over the x axis</td></tr>
<tr><td>Parent function (Graph)</td><td>
<u>Transformed Exponential function</u>


</td></tr>
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<table>
<tr>
<td>Parent function (Table)</td>
<td>
<table>
<tr><th>x</th><th>$y=2^x$</th></tr>
<tr><td>-3</td><td>$\frac{1}{8}$</td></tr>
<tr><td>-2</td><td>$\frac{1}{4}$</td></tr>
<tr><td>-1</td><td>$\frac{1}{2}$</td></tr>
<tr><td>0</td><td>1</td></tr>
<tr><td>1</td><td>2</td></tr>
<tr><td>2</td><td>4</td></tr>
</table>
</td>
</tr>
<tr>
<td>Function Characteristics</td>
<td>Characteristics (f(x) = 2^x)
Domain: {all real numbers}
Range: {f(x)| f(x) > 0}
Zeros: none
x-intercepts: none
y-intercepts: (0, 1)
Asymptote: y = 0
End Behavior: As x approaches ∞, f(x) approaches + ∞. As x approaches - ∞, f(x) approaches 0.</td>
</tr>
<tr>
<td>Geometric Sequence and Series</td>
<td><u>Geometric Sequence</u>
Recursive
$t_n = t_{n-1} \times r^n$
t_n is the nth term of the sequence
t_{n-1} is the previous term of the nth term of the sequence
r is the common ratio
n is the term number

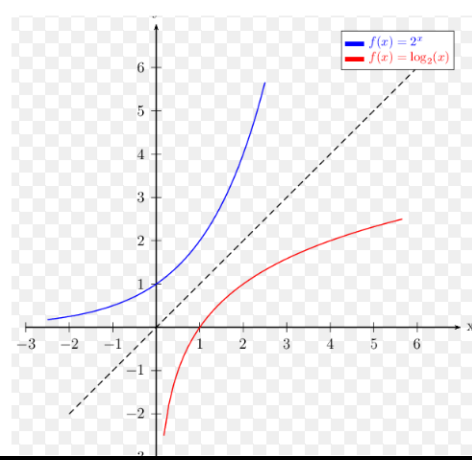
Explicit
$t_n = t_1 \times r^{n-1}$

t_n is the nth term of the sequence
t_1 is the first term
r is the common ratio
n - 1 is the 1 less than the term number
n is the term number

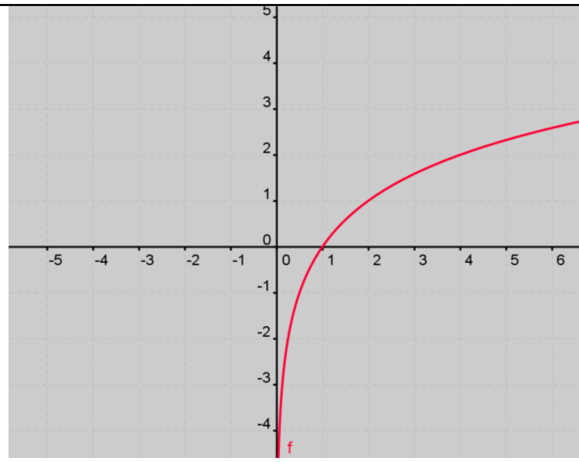
<u>Geometric series</u>
$S_n = t_1 \frac{(1-r^n)}{(1-r)}$

<u>Infinite Series</u>
$S_n = \frac{a(r^n-1)}{r-1}$

S_n is the sum of n terms
a is the initial term
r is the common ratio
r - 1 is the 1 less than the common ratio
n is the term number</td>
</tr>
</table>

	Logarithmic function
Verbal description	Inverse of the exponential function
Real Life	<u>**Inverse Function**</u> If you wanted to know how many years it will take for your \$2000.00 to double with a 5% interest then you would use logarithm(inverse of exponential) to answer this question for example $4000 = 2000 \times 1.05^t$ $2 = 1.05^t$ How many times do you need to multiply 1.05 by itself to get 2? You can take log on both sides to answer this question Log2/log1.05 = t **Homemade chocolate chip cookies can lose their freshness over time. When the cookies are fresh, the taste quality is 1. The taste quality decreases according to the function:** ***f*(*x*) = 0.8^x, where *x* represents the number of days since the cookies were baked and *f(x)* measures the taste quality. When will the cookies taste half as good as when they were fresh?**  $0.5 = 0.8^x$ log 0.5 = x log 0.8 x = log 0.5 ÷ log 0.8 x = 3 days
Function form	<u>**Inverse exponential function**</u> $f(x) = b^p$ $f^{-1}(x) = log_{\,b} f(x)$ where b>0 and b ≠ 1 b base p is the power $f^{-1}(x)$ is the inverse function 

Parent function (Graph)



Parent function (Table)

x	$y = \log_2 x$
$\frac{1}{8}$	-3
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2

Function Characteristics

Domain: {x| x > 0}

Range: {all real numbers}

Zeros: x=1

x-intercepts: (1, 0)

y-intercepts: none

Asymptotes: $x = 0$

End Behavior: As x approaches ∞, y approaches + ∞**.**

Change of base

$$Log_b x = \frac{Logb}{logx}$$

Properties of logarithms

1. $\log_a (uv) = \log_a u + \log_a v$	1. $\ln (uv) = \ln u + \ln v$
2. $\log_a (u / v) = \log_a u - \log_a v$	2. $\ln (u / v) = \ln u - \ln v$
3. $\log_a u^n = n \log_a u$	3. $\ln u^n = n \ln u$

Definition of natural logarithm

When $e^y = x$ then $Log_e x = y$ or lnx = y, where e ≈ *2.71828183*

And $Log_e e = 1$ or lne = 1 because $e^1 = e$

PARCC Sample Item

CCSS.MATH.CONTENT.HSA.SSE.B.3.C

Use the properties of exponents to transform expressions for exponential functions. ***For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.***

A bank pays depositors a 2% interest rate compounded semiannually. Let P represent an initial deposit and let t represent the number of years that the deposit is in the bank. The expression $P\left(1+\frac{0.02}{2}\right)^{2t}$ can be used to determine the account balance after t years. Which expression accurately reflects the annual interest rate?

A. $P(1.01)^t$

B. $P(1.21)^t$

C. $P(1.0201)^t$

D. $P(1.0404)^t$

CCSS.MATH.CONTENT.HSF.LE.A.4

For exponential models, express as a logarithm the solution to *ab^{ct}* = *d* where *a, c,* and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.

$$Log_2 6 \cong 2.6$$

Use this to solve for x in the equation $12 = 8^x$

CCSS.MATH.CONTENT.HSF.BF.B.5

(+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

In the overview you developed function rules that modeled the depreciation of two cars. When will the value of both cars be the same?

Car 1: $20,000(0.92)^t$ **Car 2:** $12 = 16,000(0.93)^t$

CCSS: N.RN.2

CCSS N.RN.2: Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Task 1:

For $m > 0$, the expression $\frac{2(\sqrt{m})^3}{\sqrt[4]{m}}$ can be rewritten in the form $2m^a$, where a is a fraction.

What is the value of a?

Enter your answer in the boxes.

$$\frac{\boxed{}}{\boxed{}}$$

Task 2

If $\sqrt{\sqrt[3]{(x+1)^5}} = (x+1)^a$, for $x \geq -1$, and a is a constant, what is the value of a?

Ⓐ $\frac{3}{10}$

Ⓑ $\frac{5}{6}$

Ⓒ $\frac{5}{3}$

Ⓓ $\frac{10}{3}$

Task 3:

Which expression is equivalent to $(\sqrt[3]{27})^4$?

Ⓐ 12

Ⓑ 9^2

Ⓒ 81^4

Ⓓ $27^{\frac{3}{4}}$

Task 4:

Rewrite the expression $9\sqrt[5]{27}$ as a power of 3.

Rewrite the expression $\sqrt[3]{x^2}$ in exponential form.

Which of the following is equivalent to $a^{\frac{1}{2}}b^{\frac{3}{4}}$?

a. $\sqrt{ab^3}$

b. $\sqrt{a^2b^3}$

c. $\sqrt[4]{ab^3}$

d. $\sqrt[4]{a^2b^3}$

Task 5:

A company that manufactures memory chips for digital cameras uses the formula $c = 3\sqrt{n}\left(40\sqrt[6]{n} + 9\sqrt[4]{n}\right)$ to determine the cost, c, in dollars, for producing n chips. This formula can be written as $c = 120\sqrt[3]{n^a} + 27\sqrt[4]{n^b}$, where a and b are constants. What are the values of a and b?

Enter your answers in the boxes.

$a =$ ☐ , $b =$ ☐

Task 6

An expression is given.

$$\frac{(3x)}{(3x)^{\frac{3}{2}}}$$

If $x > 0$, which of the expressions listed is equivalent to the expression given?

Select **all** that apply.

- ☐ A. $\frac{1}{3x}$
- ☐ B. $\frac{1}{\sqrt{3x}}$
- ☐ C. $\frac{1}{3\sqrt{x}}$
- ☐ D. $(3x)^{\frac{1}{2}}$
- ☐ E. $(3x)^{-\frac{1}{2}}$

Task 7

Given that $x > 0$, which expression is equivalent to $5\sqrt{xy} + 25\sqrt{x}$?

Ⓐ $5(xy)^{-1} + 25x^{-1}$

Ⓑ $25x^{\frac{1}{2}}(\sqrt{y} + 5)$

Ⓒ $\sqrt{x}\left(25y^{\frac{1}{2}} + 5\right)$

Ⓓ $5x^{\frac{1}{2}}\left(y^{\frac{1}{2}} + 5\right)$

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Algebra II Unit 3
CCSS: A.SSE2
Use the structure of an expression to identify ways to rewrite it

Task 1:

Which equations are true for all values of x?

Select **all** that apply.

- ☐ A. $3^{2-x} = 3^2 - 3^x$
- ☐ B. $3^{x+2} = 9\left(3^x\right)$
- ☐ C. $\left(3^x\right)^2 = \left(3^2\right)^x$
- ☐ D. $9^{x+2} = 3^{2x+4}$
- ☐ E. $27^x = \left(3^x\right)^3$

Task 2:

Solve the equation $27^x = 9^{x-3}$ for x.

Task 3

Consider the equation $\frac{4^{x^2}}{2^x} = 2$.

4. Part A

Which equation is equivalent to the equation shown?

Ⓐ $2^{x^2} = 2$

Ⓑ $2^{x^2 - x} = 2$

Ⓒ $2^{2x} = 2$

Ⓓ $2^{2x^2 - x} = 2$

Part B

Which values are solutions to the equation?

Select **all** that apply.

Ⓐ -2

Ⓑ -1

Ⓒ $-\frac{1}{2}$

Ⓓ $\frac{1}{2}$

Ⓔ 1

Ⓕ 2

Task 4

Consider the equation $(x^2 + 2xy + y^2)(x + y) = 64$.

Part A

What is the value of $x + y$?

Ⓐ 2

Ⓑ 4

Ⓒ 8

Ⓓ 32

Part B

If $z > 0$ and $z^x z^y = 81$, what is the value of z?

Task 5

Consider the expression $3^x - 3^{x-2}$.

Part A

Which is an equivalent form of the given expression?

Ⓐ $3^x - 9(3^x)$

Ⓑ $3^x - 2(3^x)$

Ⓒ $3^x - \frac{3^x}{2}$

Ⓓ $3^x - \frac{3^x}{9}$

Part B

This expression can also be rewritten in the form $a(3^x)$, where a is a constant. What is the value of a?

Ⓐ $\frac{1}{9}$

Ⓑ $\frac{1}{2}$

Ⓒ $\frac{8}{9}$

Ⓓ $\frac{3}{2}$

Task 6

If $\left(7^{\frac{1}{4}}\right)^{x} = 7$, what is the value of x ? Explain your reasoning.

Task 7

The function $f(x) = 8^{2x+3}$ is equivalent to the function $f(x) = 2^{a}b^{x}$, where a and b are integers. What are the values of a and b ?

Task 8

Rewrite the function $g(x) = 10^{4}\left(2^{3x}\right)\left(0.1^{x}\right)$ in the form $g(x) = n(b)^{x}$, where n and b are constants.

Task 9:

Which of the following functions is equivalent to $f(x) = 3^{x} \cdot 2^{3x+2}$?

a. $f(x) = 2(24)^{x}$

b. $f(x) = 2(6)^{3x^2}$

c. $f(x) = 4(24)^{x}$

d. $f(x) = 4(6)^{3x^2}$